

# TOWARDS MEASURING INTONATION QUALITY OF CHOIR RECORDINGS: A CASE STUDY ON BRUCKNER’S LOCUS ISTE

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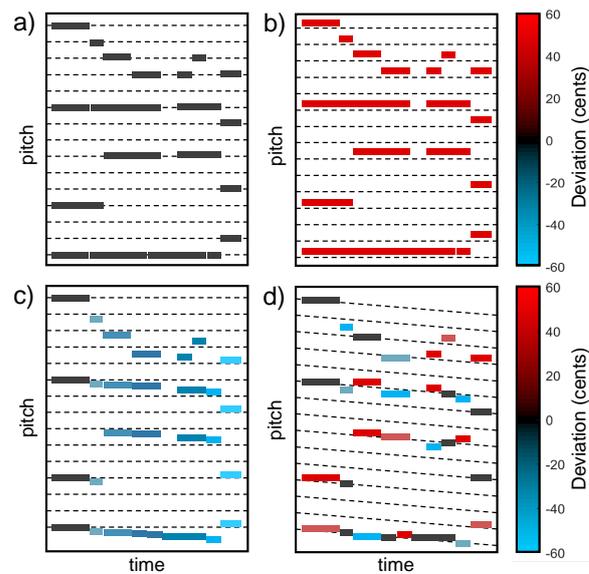
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## ABSTRACT

Unaccompanied vocal music is a central part of Western art music, yet it requires excellent skills for singers to achieve proper intonation. In this paper, we analyze intonation deficiencies by introducing an intonation cost measure that can be computed from choir recordings and may help to assess the singers’ intonation quality. With our approach, we measure the deviation between the recording’s local salient frequency content and an adaptive reference grid based on the equal-tempered scale. The adaptivity introduces invariance of the local intonation measure to global intonation drifts. In our experiments, we compute this measure for several recordings of Anton Bruckner’s choir piece *Locus Iste*. We demonstrate the robustness of the proposed measure by comparing scenarios of different complexity regarding the availability of aligned scores and multi-track recordings, as well as the number of singers per part. Even without using score information, our cost measure shows interesting trends, thus indicating the potential of our method for real-world applications.

## 1. INTRODUCTION

Unaccompanied vocal music constitutes the nucleus of Western art music and the starting point of polyphony’s evolution. Despite an increasing number of studies [1, 4, 5, 8–11, 16, 17, 21] dating back to the 1930s [29], many facets of polyphonic a cappella singing are yet to be explored and understood. A central challenge of a cappella singing is the adjustment of pitch in order to stay in tune relative to the fellow singers. Even if choirs achieve locally good intonation, they may suffer from intonation drifts slowly evolving over time [8–10, 15–17, 21, 23]. Thus, one has to deal with different intonation issues that refer to harmonic (or *vertical*) and melodic (or *horizontal*) intonation. In Fig. 1, we show how these different aspects of intonation quality may be visualized separately. Our schematic example illustrates the assessment of note-wise pitch deviations (color-coded) in the presence of a global pitch drift. Fig. 1a shows the



**Figure 1.** Note-wise analysis for polyphonic music. (a) Global intonation matching a fixed reference grid. (b) Global intonation higher than a fixed reference grid. (c) Intonation drift shown against a fixed reference grid. (d) Intonation drift with deviations from an adaptive grid.

idealized situation, where each note-wise pitch lies on a fixed reference grid. In Fig. 1b, all pitches are sharp (too high) compared to the same reference grid. The intonation quality, however, should be considered equivalent in both cases. Fig. 1c illustrates a different situation with downward intonation drift. Using a fixed reference grid, the deviations gradually accumulate. To compensate for this, one can use an adaptive reference grid [12, 15, 31] so that the vertical intonation quality is separated from the horizontal intonation drift. The residual pitch deviations—relative to the adaptive grid—only refer to vertical intonation problems (Fig. 1d), which are in the focus of our analysis.

In this paper, we propose an intonation cost measure that can be computed from choir recordings and may help to assess the singers’ intonation quality. Developing such a measure encompasses two central challenges: (i) accurate estimation of the local salient frequency content from a choir recording, and, (ii) reliable measurement of intonation quality corresponding to human perception on the one hand and to music theory on the other hand.

Concerning the estimation of the local salient frequency content (i), recordings of polyphonic choir pieces constitute extremely difficult scenarios. Often, the different

parts of a musical composition are highly correlated both in rhythm (joint on- and offsets) and harmony (overlap of partials). In the case of a *mix* recording (single-track), this leads to overlaps in time–frequency representations, which makes the estimation of fundamental frequencies (F0s) [3] or partial tracking [25] hard. On the other hand, capturing intonation at the sub-semitone level requires high frequency resolutions. One can leverage these problems using dedicated recording scenarios where singers are isolated acoustically [5] or recorded sequentially [4]. Alternatively, special devices such as Larynx microphones [6, 16, 28] or additional score information [9] can help to simplify the F0-estimation problem [18, 27]. In Section 3.2, we detail the strategy used for this paper’s experiments.

For assessing intonation quality (ii), we aim towards developing a robust intonation cost measure. Ideally, a high value of this measure indicates passages of low intonation quality. Hereby, intonation quality may relate to human perception such as the measure proposed in [30] based on psychometric curves [26]. On the other hand, intonation quality may be guided by music theory or historical performance practice [9]. In particular, choirs often aim for just intonation, which involves complex adjustment strategies according to the harmonic context [10]. In contrast to such ideas, we follow a simplified approach based on 12-tone equal temperament (12-TET). Even though 12-TET is not considered to be the ideal intonation practice for Western choir performance, it is a first approximation and can provide useful feedback [15]. As a major advantage, our strategy estimates the sub-semitone intonation quality for *any* type of notated chord irrespective of its *harmonic* consonance—in contrast to other methods [30] that measure a mixture of harmonic consonance and intonation.

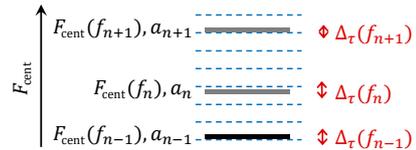
As our main contribution, we propose a 12-TET-based intonation cost measure (Section 2). Inspired by prior work [15, 24], we accumulate the overall deviation of frequency components from an adaptive 12-TET grid weighted by their corresponding amplitudes. For testing this measure, we compiled a small but diverse dataset of Anton Bruckner’s *Graduale Locus Iste* using different performances (Section 3). We evaluate the robustness of our method by comparing scenarios of varying complexity regarding the availability of aligned scores and multi-track recordings (Section 4.1). Finally, we apply our method to different performances and show its benefit for assessing the overall intonation quality of a recording (Section 4.2). Section 5 concludes the paper and gives an outlook on future work and practical applications.

## 2. MEASURING INTONATION QUALITY

In the following, we describe the computation of an intonation cost measure based on frequency deviations from a 12-tone equal-temperament (12-TET) grid.

### 2.1 Intonation Cost Measure

The proposed measure operates on a set of  $N$  frequency components  $\mathcal{P} := \{(f_1, a_1), \dots, (f_N, a_N)\}$ , where each



**Figure 2.** Grid deviations  $\Delta_\tau(f_n)$  in cents of frequency components  $f_n$  (solid gray lines) from a 12-TET grid (dashed blue lines) shifted by  $\tau$ . The corresponding amplitudes  $a_n$  are indicated by the grayscale colors.

tuple  $(f_n, a_n) \in \mathcal{P}$  denotes the frequency  $f_n$  and amplitude  $a_n$  of an individual component.

First, we convert a given frequency  $f$  in Hertz (Hz) to cents by

$$F_{\text{cent}}(f) := 1200 \cdot \log_2 \left( \frac{f}{f_0} \right), \quad (1)$$

where  $f_0 := 55$  Hz is an arbitrary but fixed reference frequency. We compute the deviation of the frequency component  $f$  from a 12-TET grid

$$\Delta_\tau(f) := \min_{i \in \mathbb{Z}} |F_{\text{cent}}(f) - \tau - 100i|, \quad (2)$$

where  $\tau \in [-50, 50[$  specifies the overall grid shift in cents, see Fig. 2. Applying a Gaussian-like function to the grid deviation  $\Delta_\tau(f)$ , we define the intonation cost (IC)  $\Theta_\tau$  as

$$\Theta_\tau(\mathcal{P}) := \frac{\sum_{(f,a) \in \mathcal{P}} a \left( 1 - \exp \left( -\frac{\Delta_\tau^2(f)}{2\sigma^2} \right) \right)}{\sum_{(f,a) \in \mathcal{P}} a}, \quad (3)$$

where the deviations are weighted and then normalized by the corresponding amplitudes. Due to the normalization, we obtain  $\Theta_\tau(\mathcal{P}) \in [0, 1]$  where  $\Theta_\tau(\mathcal{P}) = 1$  indicates the maximal IC. The parameter  $\sigma$  adjusts the cost for deviating from the grid. As suggested by [24], we choose a value of  $\sigma = 16$  cents. To obtain invariance to pitch drifts, i.e., variation of the reference frequency, we choose the grid shift  $\tau$  in an adaptive way so that the IC is minimized:

$$\tau^* := \arg \min_{\tau \in [-50, 50[} \Theta_\tau(\mathcal{P}). \quad (4)$$

We then define the intonation cost  $\Theta$  as

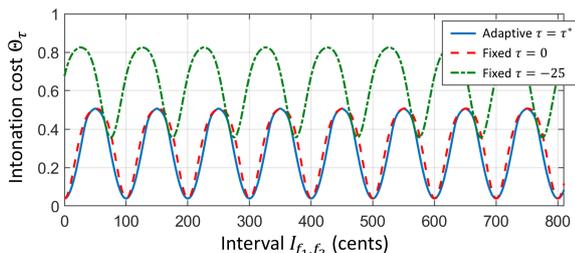
$$\Theta(\mathcal{P}) := \Theta_{\tau^*}(\mathcal{P}). \quad (5)$$

For instance, in a scenario where a choir performs with good local intonation but is affected by a pitch drift,  $\tau^*$  slowly changes over time while  $\Theta$  is constantly small.

### 2.2 Example with Synthetic Signals

In the following, we illustrate the properties of the IC measure  $\Theta(\mathcal{P})$  by means of synthetic examples. To this end, we define a harmonic tone  $x_f : \mathbb{R} \rightarrow \mathbb{R}$  with  $K$  partials and fundamental frequency  $f$  as

$$x_f(t) := \sum_{k=1}^K a_k \cdot \sin(2\pi kft), \quad (6)$$



**Figure 3.** ICs  $\Theta_\tau$  for two harmonic tones  $x_{\text{duad}}$  with an interval of size  $I_{f_1, f_2}$  in cents and three grid shifts: adaptive grid  $\tau^*$ , fixed grid  $\tau = 0$ , and, fixed grid  $\tau = -25$ . The F0 of the lower tone is fixed at  $f_1 = 220$  Hz.

where  $t$  denotes time and

$$a_k := s^{k-1} \quad (7)$$

denotes the geometrically decaying partial amplitudes for some  $s \in [0, 1]$  and  $k = 1, \dots, K$ . Thus, the set of frequency components of signal  $x_f$  is

$$\mathcal{P}[x_f] := \{(f, a_1), (2f, a_2), \dots, (Kf, a_K)\}. \quad (8)$$

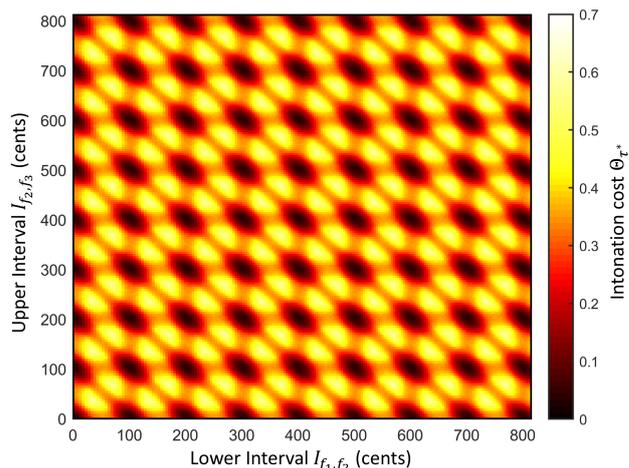
Let  $x_{\text{duad}} := x_{f_1} + x_{f_2}$  be the sum of two harmonic tones whose fundamental frequencies differ by the interval

$$I_{f_1, f_2} := |F_{\text{cent}}(f_2) - F_{\text{cent}}(f_1)| \quad (9)$$

given in cents. Consequently,  $\mathcal{P}[x_{\text{duad}}] = \mathcal{P}[x_{f_1}] \cup \mathcal{P}[x_{f_2}]$ .

Fig. 3 shows  $\Theta(\mathcal{P}[x_{\text{duad}}])$  for two harmonic tones with  $K = 16$  and  $s = 0.6$  for different interval sizes  $I_{f_1, f_2}$ . The lower fundamental frequency is set to  $f_1 = 220$  Hz such that  $\Delta_0(f_1) = 0$ .  $\Theta(\mathcal{P}[x_{\text{duad}}])$  is minimal for  $I_{f_1, f_2}$  being an integer multiple of 100 cents. However,  $\Theta(\mathcal{P}[x_{\text{duad}}])$  does not reach zero as some partial frequencies  $k \cdot f$  of a harmonic tone do not lie on the 12-TET grid even if the fundamental frequency  $f$  does. For example, the third partial  $3f_1 = 660$  Hz leads to  $\Delta_0(3f_1) \approx 2$  cents and the fifth partial  $5f_1 = 1100$  Hz leads to  $\Delta_0(5f_1) \approx 14$  cents. Since the minimal values are close to zero, this effect is small for a partial decay of  $s = 0.6$ . On the other hand, even a quarter tone interval  $I_{f_1, f_2} = 50$  cents does not lead to the maximal IC of 1, since the grid deviation  $\Delta_{\tau^*}(k \cdot f_1) \approx \Delta_{\tau^*}(k \cdot f_2) \approx 25$  cents for  $\tau^* = 25$  cents. Fig. 3 further shows that the IC of a fixed grid shift  $\tau = 0$  is similar to the adaptive grid  $\tau^*$ , while a fixed shift  $\tau = -25$  significantly increases the overall IC. It is important to note that the IC with adaptive grid shift  $\tau^*$  is invariant to the choice of  $f_1$  while a fixed grid shift is not. Further, the minimal and maximal values depend on the amplitude decay  $s$  and on the Gaussian width  $\sigma$ . Since the IC measure relates to a 12-TET grid, the IC curve is periodic in interval size with a period of 100 cents. As a consequence, each musical interval is only evaluated by its deviation from the 12-TET scale—regardless of its harmonic consonance quality.

In Fig. 4, we expand the previous example to three harmonic tones  $x_{\text{triad}} := x_{f_1} + x_{f_2} + x_{f_3}$  with  $f_1 \leq f_2 \leq f_3$ . For instance, the intervals  $I_{f_1, f_2} = 400$  cents,  $I_{f_2, f_3} = 300$  cents describe an equal-tempered major triad. The colors in Fig. 4 indicate  $\Theta(\mathcal{P}[x_{\text{triad}}])$  with respect to the



**Figure 4.** IC  $\Theta$  for three harmonic tones  $x_{\text{triad}}$ . The plot axes indicate the size of the lower and upper interval in cents,  $I_{f_1, f_2}$  and  $I_{f_2, f_3}$  respectively.

lower and upper intervals,  $I_{f_1, f_2}$  and  $I_{f_2, f_3}$ , respectively. Similar to Fig. 3, we observe a periodic structure with period 100 cents as the IC is invariant to the musical intervals. Thus, the measure is equally suited for estimating the intonation quality of both consonant and dissonant triads.

### 3. EXPERIMENTAL SCENARIO

This section describes the experimental scenario for applying our intonation cost measure to choir recordings.

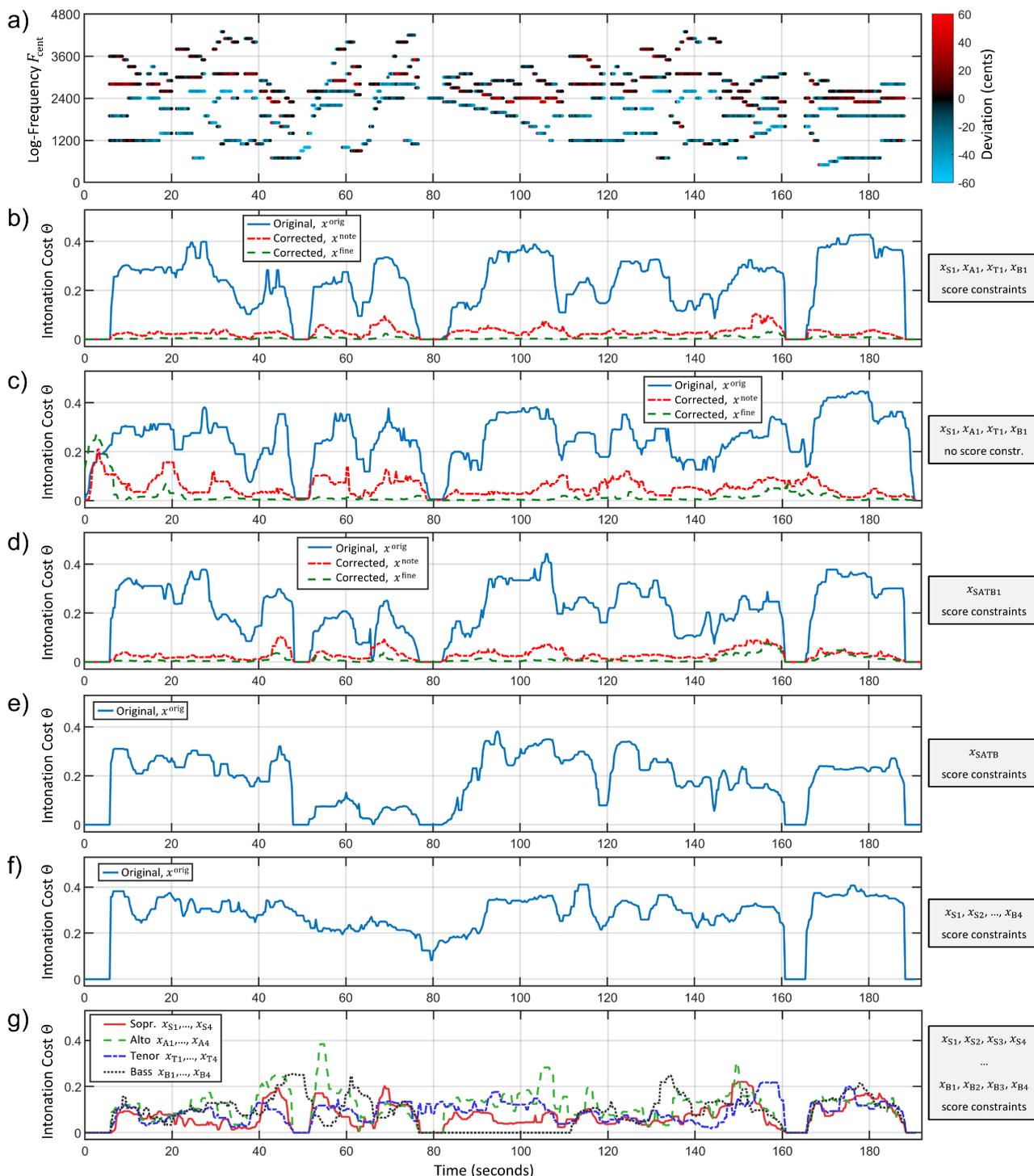
#### 3.1 Dataset

We compiled a small but diverse dataset of performances of Anton Bruckner's Gradual *Locus iste* WAB 23 (see Fig. 5). This choir piece is in Latin and lasts approximately three minutes. It is musically interesting, contains several melodic and harmonic challenges—such as the highly chromatic middle part—but also harmonically clear passages, and covers a large part of each voice's tessitura.

Central to this dataset is a publicly available<sup>1</sup> multi-track recording from the Choral Singing Dataset (CSD) [4]. The performance of 16 singers from the semi-professional Anton Bruckner choir (Barcelona) was recorded in a studio setting. The four musical parts—soprano, alto, tenor, and bass—were recorded sequentially using directional hand-held microphones. Rhythmic and harmonic synchronization was ensured by a conducting video and an acoustic reference (MIDI version of the piece). Due to this recording scenario, the individual singers' tracks exhibit a small amount of bleeding from other singers of the same part (e.g., soprano 2, 3, and, 4 slightly bleed into soprano 1 track). Interactive intonation or adaptation across musical parts was not possible since the parts were recorded in isolation and each singer listened to the reference MIDI signal while singing—this also prevented substantial pitch drifts. The restricted interaction limits the usability of the data to study intonation and

<sup>1</sup> <https://zenodo.org/record/1319597#.XJor8ShKhaR>





**Figure 6.** Intonation cost (IC) curves for different CSD versions of *Locus Iste*. (a) Piano roll representation with a log-frequency axis where C2 corresponds to 1200 cents, C3 to 2400 cents etc. The colors encode the deviation of the first voice of each part ( $x_{S1}, x_{A1}, x_{T1}, x_{B1}$ ) from the MIDI reference. (b) IC curves for  $x_{S1}, x_{A1}, x_{T1}, x_{B1}$  with score constraints from four individual tracks. The blue curve corresponds to  $x^{\text{orig}}$ , the red curve to  $x^{\text{note}}$ , and the green dashed curve to  $x^{\text{fine}}$ . (c) IC curves as in (b) without score constraints. (d) IC curves for mixed signal  $x_{SATB1}$  with score constraints. (e) IC curve for mixed signal of all voices  $x_{SATB}$  with score constraints. (f) IC curve computed from all 16 individual tracks  $x_{S1}, x_{S2}, \dots, x_{B4}$  with score constraints. (g) Individual IC curves for the four tracks of each part with score constraints.

IC increases, thus indicating that the singers have difficulties to stay in tune in this passage.

As a sanity check, we compare the results for  $x^{\text{orig}}$  to the pitch-corrected versions  $x^{\text{note}}$  and  $x^{\text{fine}}$ . As we expected, the note-wise corrections  $x^{\text{note}}$  (red curve) obtain

lower values than  $x^{\text{orig}}$ —with only minor peaks that may be caused by pitch fluctuations during a note event. If we correct such local fluctuations as in  $x^{\text{fine}}$  (green, dashed curve), the IC is almost constantly zero.

In Fig. 6c, we repeat the experiment of Fig. 6b without using score constraints. In this case, F0-estimation is less reliable and, in particular, confusions between F0 and higher partials may occur. However, since many partials lie on the same 12-TET grid as the F0 (octave-related partials) or very close to that (2 cents for fifth-related partials), the IC measure is largely invariant to such confusions. The high similarity between Fig. 6b and Fig. 6c confirms the IC measure’s robustness to F0-extraction errors. Only for silent passages (e. g., after 160 sec or at the beginning), we obtain higher IC values due to erroneously extracted frequency components.<sup>6</sup> We conclude that our strategy is, in principle, applicable for any multi-track recording and does not necessarily require score information.

To test the applicability in absence of multi-track recordings, we compute the IC curve from the mix signal  $x_{SATB1}$  using score constraints (Fig. 6d). Due to the constraints, confusions with higher partials cannot occur, but partials of other parts may leak into the same constraint regions, thus affecting the estimated F0s and partials’ amplitudes. The comparison of Fig. 6b and Fig. 6d indicates that such phenomena only slightly affect the IC measure.

Next, we measure the IC from the CSD’s full recording  $x_{SATB}$  (all 16 voices). F0-estimation is more challenging since the four singers of each part contribute to the same constraint regions. The resulting ICs (Fig. 6e) exhibit slightly lower values than in previous cases. We assume that having several voices per part stabilizes the F0-estimation to some degree. Overall, we see a similar trend between Fig. 6d and Fig. 6e. This might be an effect of mutual influence between the singers of each part. To further investigate the multiple-singer effect, we show in Fig. 6f the IC curves computed from all 16 individual tracks  $x_{S1}, x_{S2}, \dots, x_{B4}$ . We obtain higher IC values than in Fig. 6e, especially for the monophonic passages (e. g., at 48 sec). Due to the score constraints, this must be caused by deviations between the singers of a part, sometimes denoted as *dispersion* [4]. To analyze this, we repeat the experiment for each part separately (Fig. 6g). The resulting curve supports our hypothesis since. For instance, the high value at around 48 sec (Fig. 6f) is mainly caused by the basses’ dispersion (dotted curve in Fig. 6g).

#### 4.2 Global Analysis of Different Performances

Since our experiments on the CSD have shown the robustness of our method even for mix recordings, we finally compare the global IC values of multiple synthetic and real performances of *Locus Iste*. We align the MIDI file to all performances [13], use the resulting constraint regions for extracting F0-trajectories [27], and compute the IC curves. In Fig. 7, we show the statistics of the entire curves over time (median, mean, and standard deviation). First, we report values for sonifications using harmonic tones as defined in Eq. (6). To simulate detuning, we shifted each note’s fundamental frequency by a random value sampled from a Gaussian distribution with variance

<sup>6</sup> This problem could be leveraged using a suitable algorithm for voice activity detection [19] or on-/offset detection in vocal music [6].

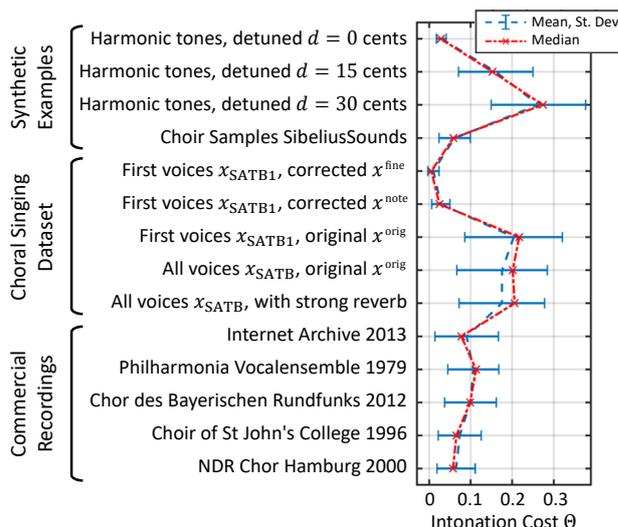


Figure 7. Statistics of the intonation cost measure  $\Theta$  over full recordings of different type.

$d^2$ . For  $d = 0$  cents, the mean IC is almost zero. For  $d = 15$  cents and  $d = 30$  cents, the IC gradually increases as expected. The average IC of a sample-based sonification<sup>7</sup> is moderately higher than the rendition with ideal harmonic tones, which suggests that some choir effects such as dispersion are also synthesized. Both corrected versions of the CSD show very low IC values, with  $x^{fine}$  even lower than  $x^{note}$ . In contrast, the versions based on the original recording  $x^{orig}$  have high IC values, where the difference between first voices only ( $x_{SATB1}$ ) and the full mix ( $x_{SATB}$ ) as well as the effect of adding artificial reverberation is marginal. All commercial recordings performed by professional choirs obtain lower IC than the CSD recording. Several effects might contribute to this observation. Besides the singers’ level of training, commercial recordings often feature larger choirs (60 singers or more) and long natural reverberation (recorded in churches). In line with the authors’ subjective judgment, the recording by NDR Chor Hamburg exhibits the best intonation quality according to the IC measure.<sup>8</sup> Overall, this experiment indicates that the proposed IC measure may serve as a first indicator for a choir recording’s global intonation quality.

#### 5. CONCLUSIONS

In this paper, we proposed a strategy for measuring the intonation quality of choir recordings. Although robust extraction of salient frequencies is challenging, the measure produced meaningful and reliable results once multi-track recordings are available or score information can be utilized. Even though the insights from such a measure are limited, it might be a first indicator for the overall intonation quality and, thus, could be useful for choir singers or choir directors in performances and rehearsals.

<sup>7</sup> Using Sibelius Sounds, see <https://www.avid.com/sibelius>

<sup>8</sup> A 30-seconds mp3 thumbnail of this recording is available at [https://www.carusmedia.com/images-intern/medien/80/8346600/8346600.010s.tl\\_010.mp3](https://www.carusmedia.com/images-intern/medien/80/8346600/8346600.010s.tl_010.mp3)

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