BISTATE REDUCTION AND COMPARISON OF DRUM PATTERNS

Olivier Lartillot

RITMO Centre for Interdisciplinary Studies in Rhythm, Time and Motion, University of Oslo olivier.lartillot@imv.uio.no

ABSTRACT

This paper develops the hypothesis that symbolic drum patterns can be represented in a reduced form as a simple oscillation between two states, a Low state (commonly associated with kick drum events) and a High state (often associated with either snare drum or high hat). Both an onset time and an accent time is associated to each state. The systematic inference of the reduced form is formalized. This enables the specification of a rhythmic structural similarity measure on drum patterns, where reduced patterns are compared through alignment. The two-state representation allows a low computational cost alignment, once the complex topological formalization is fully taken into account. A comparison with the Hamming distance, as well as similarity ratings collected from listeners on a drum loop dataset, indicates that the bistate reduction enables to convey subtle aspects that goes beyond surface-level comparison of rhythmic textures.

1. INTRODUCTION

One of the most fundamental areas of both Music Information Retrieval (MIR) and music cognition research concerns the modelling of musical similarity, due to the essential importance of similarity in music perception and the practical utility of similarity models in music retrieval applications such as recommendation systems. Modelling similarity for drum patterns is a particular challenging variant of this research, though one with potential application in a wide range of intelligent music production tools such as drum pattern recommendation systems or automatic drum pattern generation systems.

The challenges of drum pattern similarity modelling lies in the complex nature of polyphonic rhythm perception. The integration of coincident rhythms into a resultant 'multirhythm' has been argued to be an essential feature of polyphonic rhythm perception, especially in the context of drumming music [1]. Rhythmic interactions may also be a significant aspect of polyphonic rhythm perception, with complex rhythmic structures said to emerge from the interaction between rhythmic levels [2]. In the context of drum Fred Bruford Centre for Digital Music Queen Mary University, United Kingdom fred.bruford@gmail.com

patterns specifically, rhythmic interaction and instrumental configuration has been shown to have a significant affect on the perception of syncopation, a fundamental rhythmic property [3]. However, in much existing work on drum pattern similarity modelling, similarity is modelled as a linear combination of monophonic rhythm similarity models calculated on each instrument part, without consideration to rhythmic interactions or integration.

Our aim is to design new models of similarity for drum patterns that take into account both rhythmic interaction and rhythmic integration. The bistate reduction algorithm proposed in this paper attempts to extract the core structure of drum pattern as an interaction between two states, 'Low' and 'High', and to use this reduced representation to compare drum patterns through alignment.

The drum patterns used in this paper are part of a varied set of 160 stylistically accurate symbolic patterns recorded by human drummers on an electronic drum kit, taken from the Groove library of BFD3 [4], a commercially available virtual drum kit plugin. Originally collected by [5], the pattern dataset is drawn from a wide range of genres and subgenres, with 8 genre groups used: Hiphop/Dance, Funk, Blues/Country, Pop, Reggae/Latin, Rock, Metal and Jazz. In addition to multiple genres, they are of varied complexity and function, with some containing fills.

In Section 2, we discuss in greater detail approaches to modelling monophonic rhythm similarity and assess the state-of-the-art in drum pattern similarity modelling and its limitations. In Section 3 and Section 4, we begin discussion of the bistate reduction algorithm and bistate sequence alignment technique respectively as a novel means of representing complex drum patterns and estimating the perceived similarity between them. In Section 5, the sequence alignment algorithm is evaluated in its ability to predict human similarity ratings for pairs on drum patterns in our dataset.

2. RELATED WORK

Models of rhythmic similarity are significant in both music perception and music informatics research due to rhythm's fundamental importance in music. For drum patterns in particular, models of rhythmic similarity have various practical applications such as automatic generation of variations on drum patterns [6,7], or in visually mapping drum patterns by similarity to facilitate exploration of drum pattern libraries [8,9].

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Figure 1. Drum pattern *Early RnB 33*. Velocity is represented in grey level, from 0 in white to maximum value in black.



Figure 2. Reduction of *Early RnB 33*. State accents are shown in black and state onsets (if different from accents) in grey. 'Low' corresponds to \downarrow and 'high' to \uparrow .

Many models of rhythmic similarity for drum patterns are based on monophonic rhythm similarity models. These methods are based on multiple possible representations of rhythms. The most common method is as a vector consisting of either an onset or silence at each metrical position of the rhythm. The Hamming distance, a simple and popular model, works by counting the number of metrical positions in two rhythm vectors that differ in value (one onset, the other rest) [10]. The swap or directed-swap distance can also be calculated between rhythm vectors by counting the number of swap operations (exchanges between adjacent metrical positions) that need to be performed to turn one rhythm into the other [11]. Other possible rhythm similarity measures use inter-onset intervals (IOIs), vectors of the distance between each successive onset as a multiple of the smallest possible metrical position. An example of this is the chronotonic distance [10]. A thorough overview of numerous approaches to rhythm similarity modelling can be found in [10].

Typically, drum pattern similarity is modelled by stacking these monophonic rhythm similarity models calculated on each part separately. This has been found to be successful in a few cases; [7] found that the Hamming distance and directed-swap distance correlated strongly with human ratings of similarity for drum patterns, with the Hamming distance outperforming the directed-swap. In [8], the authors found a 2D projection of the Hamming distance could partially cluster drum patterns in a way that matched their genre tags for a small dataset of patterns. In a pilot study, a model similar to the Hamming distance was also found to correlate to listeners' similarity ratings for drum patterns [12]. This model counts whether the number of metrical positions where rhythms differ, but adds a weighting metric based on metrical awareness.

The limitation of these models is that by stacking monophonic rhythm similarity measures calculated on each part separately, they fail to account for effects of rhythmic interaction between instrument parts, or the perception of re-



Figure 4. Reduction of Reggae Grooves Fill 3.

sultant 'multirhythms' due to the perceptual integration of simultaneous rhythmic streams. Additionally, while fixed weighting schemes have been used to weight more perceptually important kit instruments [6], these may not be flexible enough to account for the possibly changing contextual importance of different instruments in a drum pattern.

3. BISTATE REDUCTION

Basic drum patterns are usually defined by the alternation of, typically, bass (or "kick") drum and snare drum strokes, with a further subdivision on the ride cymbal or hi-hat¹. The ordering in this definition implicitly indicates that the bass and snare drums alternation is of higher level of salience, while the cymbal or hi-hat subdivision is of lesser importance.

3.1 Dominant Drum Selection

The first hypothesis guiding the approach presented in this paper is that drum patterns can be reduced by only focusing on these two most dominant classes of drum strokes. In the definition of basic drum patterns above, the two dominant drums are said to be bass and snare drums. But sometimes the snare drums can be replaced by, for instance, closed hi-hat.

For a given drum pattern, we propose a simple method for selecting the two dominant drums. It consists in ordering the drums in a decreasing hierarchical order, and selecting the two first drums, in this ordered list, that are active in the given drum pattern. The first selected drum will be called the *Low* drum, as it often relates to the bass drum and to the use of lower frequencies, while the second selected drum will be called the *High* drum.

For the drum patterns discussed throughout the paper, the reduction was performed by using the following hierarchical ordering: 1. kick drum, 2. snare drum, 3. closed hi-hat. It was not necessary to consider other drums, since at least two of these three drums were active in each drum

¹ Cf. for instance https://en.wikipedia.org/wiki/Drum_ beat

pattern. In other words, a drum pattern featuring kick drum and closed hi-hat, but no snare drum, will use kick drum as Low and closed hi-hat as High. A drum pattern featuring snare drum and closed hi-hat but no kick drum will use snare drum as Low and closed hi-hat as High.

For the example shown in Figures 1 and 3, the closed hi-hat sequence is considered as of less importance and therefore ignored.

3.2 Two-State Alternation

The second hypothesis developed in the proposed approach is that drum patterns can be reduced as an *alternation* between the Low and High drums. This can be formalised as an alternation between two states, respectively designated \downarrow and \uparrow . Figures 2 and 4 shows the reduction corresponding to the sequences in Figures 1 and 3.

3.2.1 Handling of Simultaneous Strokes

At first sight, to detect the \downarrow and \uparrow *states*, we would simply need to look for the location of alternation between the Low and High drums. The difficulty comes from the fact that the two drums are often played together. In such occasions, inference of \downarrow or \uparrow state relies on the context defined by the recent past.

For instance in Figure 3, kick and snare drums are played together at the beginning of beats 5, 6 and 8. Because the kick drum plays a clear \downarrow at the beginning of beat 4 (i.e., 4.1), which continues with snare drum playing gently (from 4.2 to 4.4). We haven't heard any clear \uparrow state. Therefore, when snare drum and kick drums are superposed on the next beat (5.1), this is perceived as a \uparrow . But because the snare drum is played loudly on the next tick (5.2), the next beat (6.1) is rather perceived as a \downarrow . Same for 8.1.

We propose to explain this phenomenon by hypothesising that the categorisation \downarrow vs. \uparrow is based on comparing each new drum stroke (or superposed strokes) to mental models of \downarrow and \uparrow states. And in our modelling, this can be simplified by comparing to only the mental model related to one of the states \downarrow and \uparrow , namely the state considered as currently active.

More precisely, the mental model is represented by a state $s \in \{\downarrow, \uparrow, 0\}$ (where state 0 is used solely when starting the analysis, before the first actual state \downarrow or \uparrow has been detected) altogether with a series of two velocities $(v_{\downarrow}^M, v_{\uparrow}^M) \in [0, 1]^2$, called *profile* of the mental model, and first initiated to (0, 0). The profile is updated with the current configuration

$$(v_{\downarrow}^M, v_{\uparrow}^M) = (v_{\downarrow}, v_{\uparrow}) \tag{1}$$

every time there is a state transition.

At any given successive point in the drum pattern, featuring at that instant a Low drum stroke of velocity $v_{\downarrow} \in$ [0, 1] (with 0 indicating no stroke) and a High drum stroke of velocity $v_{\uparrow} \in$ [0, 1], we consider the following alternatives, which are tested in the indicated order until a suitable condition is found:



Figure 5. State transition diagrams, where the current state is respectively 0 (left diagram), \downarrow (middle) and \uparrow (right). For given values for v_{\downarrow} (X axis) and v_{\uparrow} (Y axis) is indicated the next state: \downarrow (red) \uparrow (blue), 0 (white). See the text for more explanation.

- If v_↓ > v_↑ + .2, the next state is ↓. This corresponds to the region in dark red in each diagram in Figure
 The profile is updated. In other words, when the Low stroke is significantly stronger than the High stroke, the new state is clearly ↓.
- 2. Similarly, if $v_{\uparrow} > v_{\downarrow} + .2$, the next state is \uparrow (dark blue region in Figure 5) and its profile updated.
- If the current state is 0 (i.e, undefined), the next state is ↓ if v_↓ > .2 (light red region in the left diagram of Figure 5). In other words, at the very beginning of a drum pattern, an ambiguous mixture of Low and High drums is associated with ↓, because it is considered as the default state (while ↑ is a departure from the default state). If on the contrary v_↓ ≤ .2 the next state is ↑ if v_↑ > .2 or if v_↓ = 0 (light blue region).
- 4. If $v_{\downarrow} = 0$ and if the current state is \downarrow , the next state is \uparrow if $v_{\uparrow} > \frac{v_{\uparrow}^M}{2}$. If on the contrary the current state is already \uparrow , the profile is updated only if $v_{\uparrow} > v_{\uparrow}^M$.
- 5. If the current state is \downarrow and $v_{\uparrow} > v_{\uparrow}^M + .2$, the next state is \uparrow (light blue region in the middle diagram of Figure 5). In other words, a significant increase in velocity of the High strike (even if the velocity remains lower than the Low strike's one) triggers a transition to the \uparrow state.
- Similarly, if the current state is ↑ and v_↓ > v^M_↓ + .2, the next state is ↓ (light red region in the right diagram of Figure 5).

3.2.2 State Onset and Accent Times

The approach presented in the previous section leads to the generation of a sequence of states \uparrow and \downarrow . Successive repetitions of a same state are reduced into one single state. This practically means in particular that successive repetitions of a given drum (while the other drum remains silent) are collapsed into one single state.

However, if the most accented stroke is not the first one, this needs to be specifically indicated in the pattern description, as it plays an important role in the pattern characteristics. For that reason, with each state (excepted the initial 0 state) are associated three attributes:

	↓!	(\downarrow)	^!	(\uparrow)
↓!	$S\downarrow$	$S\downarrow$	0	$A\downarrow$
(\equiv)	$S\downarrow$		$B\uparrow$	
↑!	0	$A\uparrow$	$S\uparrow$	$S\uparrow$
(†)	$B\downarrow$		$S\uparrow$	

Table 1. Alignment configurations, triggered by state transitions, in the first sequence (left column) or/and in the second sequence (top row). In the absence of transition in any of the two sequences, no alignment configuration needs to be defined.

- whether it is \downarrow or \uparrow
- the *onset time*, i.e., the temporal position of the first stroke in that successive repetition of strokes
- the *accent time*, i.e., the temporal position of the most accented stroke.

4. BISTATE SEQUENCE ALIGNMENT

The third hypothesis developed in this paper is that the perceived distance between two drum patterns can be approximated by aligning the two corresponding bistate sequences and estimating the alignment mismatches.

4.1 Alignment principle

To allow the formalisation of this alignment, the set of two states $\{\downarrow,\uparrow\}$ is further decomposed into four states $\{\downarrow!,\uparrow$ $!,(\downarrow),(\uparrow)\}$ in order to take into account all the temporal relationships:

- ↓! and ↑! corresponds to a new transition to, respectively, ↓ and ↑.
- (↓) and (↑) indicates a continuation of the corresponding states ↓ and ↑.

This leads to a 4×4 matrix of possible alignment configurations, shown in Table 1. We can distinguish three types of alignment configurations:

- The two sequences are in same state ("S"), both low ("S ↓") or both high ("S ↑").
- One sequence introduces the opposite state ahead of the other sequence: "A" if it the first sequence ahead, "B" if it the second sequence.
- The two sequences are in opposite states ("O").

The possible transitions between configurations are the following, as described in Table 1:

- Starting from the same state (for instance S ↓), then both sequences transition at the same time (for instance S ↑).
- Starting from the same state (for instance S ↓), then one sequence transitions first (for instance A ↑).



Figure 6. Alignment between two reduced drum patterns A and B. Penalties are given for each individual state, indicated with a number or 'P'. See the text for more explanation.

- From configuration A or B, if only one sequence transitions, this leads to S, while if both sequences transition at the same time, this leads to opposite states O.
- From *O*, another double-transition leads to another *O*, while a single transition leads to either *A* or *B*.

4.2 Numerical distance estimation

The proposed distance model is based on an evaluation of how well each of the two drum patterns aligns with the other. For each successive state of each drum pattern, a penalty is given if that state is not found in the other sequence around the same temporal position. Offset in the temporal positions of the state on the two sequences also contributes to the penalty.

The basic mechanisms are illustrated below using the toy example shown in Figure 6. In this first example, we assume that both sequences start at the same state (\downarrow) .

- The first state starts at the same time (t = 1) on both sequences, leading to a S ↓ configuration with 0 penalty (shown in green in the Figure).
- The next event appears first in sequence A (A ↑ at t = 2). The same transition appears next in sequence B (S ↑ at t = 3). It is thus a quasi-synchronised transition, with 1 tick delay between the two sequences. A penalty of 1 is therefore given to the new state in each sequence. (The onset of each state is shown in orange while the position of the opposite state is shown in yellow.)
- A transition B ↓ at t = 4 is followed by another transition back to S ↑. This state ↓ is deleted by giving the maximum penalty P (in red) and the new state ↑ at t = 5 (in blue) is ignored.
- A transition A ↓ at t = 6 is followed by a double transition O at t = 8, which is considered as both a 2-tick-delayed transition of B to state ↓ and as the start of a new transition A ↑. Same for the subsequent double transition O at t = 9. The new transition A ↓ being followed by a transition S ↑, that state ↓ is removed.



Figure 7. Alignment between two reduced drum patterns A and B. Boxed series of cells show states containing repeated strokes, where the accent is the rightmost cell.

 Sequence A continuing after the end of sequence B, any transition to the opposite state A ↓ (here only at t = 11) is deleted.

In Figure 7, the sequences starting in opposite states, an additional \downarrow state is added before the start of sequence B, to which is given a penalty of 4. In this example, since the next event is a \downarrow in sequence B, the previous \uparrow is removed.

The rest of Figure 7 illustrates the handling of states containing repeated strokes with the accentuated stroke appearing after the initial onset. Concerning the transition to \uparrow first by B at t = 3 and then by A at t = 5, the penalty is only 1 because it is computed between the accent in sequence B (at t = 4) and the onset of sequence A. For the same reason, the next transition to \downarrow at t = 7 is considered without penalty. While B remains in \downarrow , with its attack at t = 9, A transitions to \uparrow with an attack at t = 8, which is therefore deleted.

Each penalty is weighted with respect to the corresponding accent velocity. For instance in the case of a drum stroke of very low intensity (such as a ghost note), the contribution of this penalty to the overall distance will be low. Finally the distance between the two sequences is defined as the maximum between the two weighted sums.

5. EVALUATION

To investigate the potential use of the bistate sequence alignment algorithm for drum pattern similarity modelling, we examined its calculated distances between 80 pairs of drum patterns in our dataset in relation to human-provided perceptual similarity ratings.

5.1 Methodology

We collected similarity ratings for the drum patterns via an online listening test. The symbolic patterns were first synthesized into audio via a generic sampled drum kit. 21 listeners rated similarity for the 80 pairs of patterns using a continuous scale. Median internal consistency for all participants calculated using the ICC (2,1) for the repeated pairs was 0.85, equaling good agreement. The inter-rater agreement for the 21 listeners calculated using the same ICC (2,1) was 0.73, equally moderate-to-good agreement. The spreads of ratings for each comparison were normally

Similarity Model	r	р
Hamming Distance	0.604	2.97e-9
Hamming Distance (2 channels)	0.539	2.58e-7
Bistate Sequence Alignment	0.556	8.49e-8
min(Hamming (2 chan), Alignment)	0.606	2.65e-9
min(Hamming, Alignment)	0.692	1.21e-12

Table 2. Pearson correlation coefficient r and p-value be-tween mean similarity ratings and distance models.

distributed. More information on the collection of this dataset may be found in [5].

To test the overall extent to which the bistate sequence alignment algorithm corresponds to perceived similarity, we calculated the Pearson correlation between the distance calculated by the bistate sequence alignment algorithm and the mean of human-supplied similarity ratings. We used the established Hamming distance as a reference for evaluation, since, as indicated in Section 2, it gave the best results in previous works. The first step of our approach, presented in section 3.1, can be studied separately by also computing the Hamming distance on the two main channels of the drum patterns.

5.2 Results and Discussion

The results can be seen in Table 2. Various values for the parameter P were tried and the best results were obtained with P = 8. The Hamming distance's correlation on the complete drum patterns is slightly stronger than the bistate sequence alignment, which is itself very slightly better than the Hamming distance computed on the two main drum channels.

When combining the Hamming distance (computed on the whole drum loops) with the alignment by taking the minimum of them both, the correlation is better than the Hamming distance alone, with a correlation of 0.692 vs 0.604. This increase in correlation was found to be statistically significant (t=1.748, p=0.04). This could indicate that both these two algorithms capture fundamentally different aspects of similarity, with the Hamming distance capturing low-level similarities between rhythms, and the bistate sequence alignment capturing qualities relating to rhythmic interaction and structure.

The difference between the distances can be seen in Figure 8 where the similarity ratings of all 80 pairs are plotted alongside distances calculated by both the Hamming distance and the bistate sequence alignment algorithm.

The differences between the bistate sequence alignment algorithm and Hamming distance can be further demonstrated through viewing of some particular examples. Figure 9 shows a pair of *Soul Grooves* patterns that share very similar kick and snare drum patterns while differing on other drum tracks. Since the similarity was judged by the listeners as high, this demonstrates the interest, in particular cases such as this one, in focusing the comparison of drum patterns on the main kick and snare drums. And indeed, computing the Hamming distance on those two main channels leads to a similarly low distance value (cf. red arrow in Figure 8).



Figure 8. Comparison of the Z-score of the new proposed distance (in blue), the Hamming distance (in yellow), the Hamming distance on the two main channels (in purple) and the listeners' ratings (in red), for each of the 80 pairs of drum patterns. The red and blue arrows indicate the examples shown in, respectively, Figures 9 and 10.



Figure 9. Alignment between drum patterns *Soul Grooves* 2 (top left and center) and *Soul Grooves 31* (top right and bottom). The only penalties contributing to the distance is a 1-tick offset (in orange) and a deleted state (in red).

Figure 10 illustrates the interest of the method beyond the simple selection of two main channels. In this example, the two sequences differ significantly at the surface level, even if we restrict the scope on the kick and snare drums. On the other hand, the corresponding reduced bistate sequences are similar and show a clear alignment, leading to a relatively low distance on par with the listeners' jugement.

6. CONCLUSIONS

This article proposed an attempt to reduce drum patterns into an underlying core structure with the aim to compare drum patterns by aligning their reduced representation one with each other. Clearly the problem of rhythmic reduction is of high complexity, and is addressed here solely as an intermediary step for the establishment of a distance measure between drum patterns. These mechanisms of selection of dominant drums and of inference of Low and High states could be studied further in order to provide more advanced description of drum patterns.



Figure 10. Alignment between drum patterns *Batucara 3* and *Cascara 5* using the same conventions as in Figure 9.

As it could have been expected, the comparison between the proposed distance and a simple surface-level Hamming distance concerning their abilities to mimic listeners similarity judgments shows that for the most part, listeners rely on surface characteristics of the overall rhythmic texture to compare drum patterns. However, in particular cases the underlying core structure can be of importance as well, and this is where the proposed distance can be of interest. Combining surface-level and deeperlevel representations seems to improve the overall similarity modelling. We may hypothesise that further progress in this endeavour could be made possible through the integration of more refined cognitive models.

7. ACKNOWLEDGEMENTS

This work was partially supported by the Research Council of Norway through its Centers of Excellence scheme, project number 262762, the MIRAGE project, grant number 287152 and the TIME project, grant number 249817. The work was also funded by the UK Engineering and Physical Sciences Research Council (EPSRC) and by a Short-Term Scientific Mission grant provided by NordForsk's Nordic University Hub "Nordic Sound and Music Computing Network—NordicSMC", project number 86892.

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