DISCOVERING TYPICAL MOTIFS OF A *RĀGA* FROM ONE-LINERS OF SONGS IN CARNATIC MUSIC

Shrey Dutta
Dept. of Computer Sci. & Engg.
Indian Institute of Technology Madras
shrey@cse.iitm.ac.in

Hema A. Murthy
Dept. of Computer Sci. & Engg.
Indian Institute of Technology Madras
hema@cse.iitm.ac.in

ABSTRACT

Typical motifs of a *rāga* can be found in the various songs that are composed in the same *rāga* by different composers. The compositions in Carnatic music have a definite structure, the one commonly seen being pallavi, anupallavi and charanam. The *tala* is also fixed for every song.

Taking lines corresponding to one or more cycles of the pallavi, anupallavi and charanam as one-liners, one-liners across different songs are compared using a dynamic programming based algorithm. The density of match between the one-liners and normalized cost along-with a new measure, which uses the stationary points in the pitch contour to reduce the false alarms, are used to determine and locate the matched pattern. The typical motifs of a *rāga* are then filtered using compositions of various *rāga*s. Motifs are considered typical if they are present in the compositions of the given *rāga* and are not found in compositions of other *rāga*s.

1. INTRODUCTION

Melody in Carnatic music is based on a concept called *rāga*. A *rāga* in Carnatic music is characterised by typical phrases or motifs. The phrases are not necessarily scale-based. They are primarily pitch trajectories in the time-frequency plane. Although for annotation purposes, *rāga*s in Carnatic are based on 12 srutis (or semitones), the *gamakā*s associated with the same semitone can vary significantly across *rāga*s. Nevertheless, although the phrases do not occupy fixed positions in the time-frequency (t-f) plane, a listener can determine the identity of a *rāga* within few seconds of an *ālāpana*. An example, is a concert during the "music season" in Chennai, where more than 90% of the audience can figure out the *rāga*. This despite the fact that more than 80% of the audience are nonprofessionals. The objective of the presented work is to determine typical motifs of a *rāga* automatically. This is obtained by analyzing various compositions that are composed in a particular *rāga*. Unlike Hindustani music, there is a huge

repository of compositions that have been composed by a number of composers in different *rāga*s. It is often stated by musicians that the famous composers have composed such that a single line of a composition is replete with the motifs of the *rāga*. In this paper, we therefore take one-liners of different compositions and determine the typical motifs of the *rāga*.

Earlier work, [9, 10], on identifying typical motifs depended on a professional musician who sung the typical motifs for that *rāga*. These typical motifs were then spotted in *ālāpana*s which are improvisational segments. It was observed that the number of false alarms were high. High ranking false alarms were primarily due to partial matches with the given query. Many of these were considered as an instance of the queried motif by some musicians. As alapana is an improvisational segment, the rendition of the same motif could be different across alapanas especially among different schools. On the other hand, compositions in Carnatic music are rendered more or less in a similar manner. Although the music evolved through the oral tradition and fairly significant changes have crept into the music, compositions renditions do not vary very significantly across different schools. The number of variants for each line of the song can vary quite a lot though. Nevertheless, the meter of motifs and the typical motifs will generally be preserved.

It is discussed in [15] that not all repeating patterns are interesting and relevant. In fact, the vast majority of exact repetitions within a music piece are not musically interesting. The algorithm proposed in [15] mostly generates interesting repeating patterns along with some non-interesting ones which are later filtered during post processing. The work presented in this paper is an attempt from a similar perspective. The only difference is that typical motifs of *rāga*s need not be interesting to a listener. The primary objective for discovering typical motifs, is that these typical motifs can be used to index the audio of a rendition. Typical motifs could also be used for *rāga* classification. The proposed approach in this work generates similar patterns across one-liners of a *rāga*. From these similar patterns, the typical motifs are filtered by using compositions of various *rāga*s. Motifs are considered typical of a *rāga* if they are present in the compositions of a particular *rāga* and are NOT found in other *rāga*s. This filtering approach is similar to anti-corpus approach of Conklin [6, 7] for the discovery of distinctive patterns.
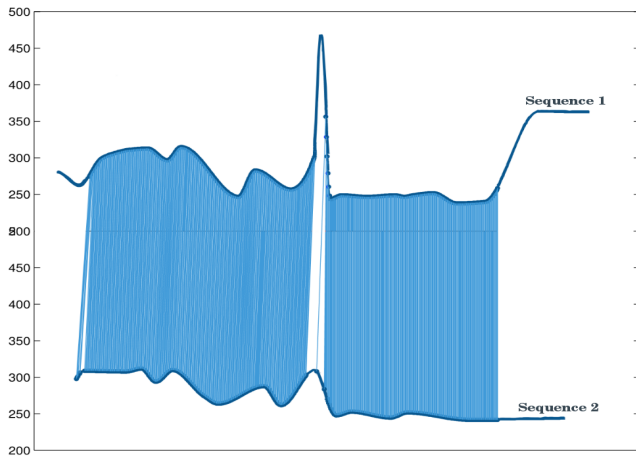
Figure 1. RLCS matching two sequences partially

Most of the previous work, regarding discovery of repeated patterns of interest in music, is on western music. In [11], B. Jansen *et al* discusses the current approaches on repeated pattern discovery. It discusses string based methods and geometric methods for pattern discovery. In [14], Lie Lu *et al* used constant Q transforms and proposed a similarity measure between musical features for doing repeated pattern discovery. In [15], Meredith *et. al.* presented Structure Induction Algorithms (SIA) using a geomatric approach for discovering repeated patterns that are musically interesting to the listener. In [4, 5], Collins *et. al.* introduced improvements in Meredith's Structure Induction Algorithms. There has also been some significant work on detecting melodic motifs in Hindustani music by Joe Cheri Ross *et. al.* [16]. In this approach, the melody is converted to a sequence of symbols and a variant of dynamic programming is used to discover the motif.

In a Carnatic music concert, many listeners from the audience are able to identify the *rāga* at the very beginning of the composition, usually during the first line itself — a line corresponds to one or more tala cycles. Thus, first lines of the compositions could contain typical motifs of a *rāga*. A pattern which is repeated within a first line could still be *not* specific to a *rāga*. Whereas, a pattern which is present in most of the first lines could be a typical motif of that *rāga*. Instead of just using first lines, we have also used other one-liners from compositions, namely, lines from the pallavi, anupallavi and charanam. In this work, an attempt is made to find repeating patterns *across* one-liners and *not* within a one-liner. Typical motifs are filtered from the generated repeating patterns during post processing. These typical motifs are available online [1]

The length of the typical motif to be discovered is not known a priori. Therefore there is a need for a technique which can itself determine the length of the motif at the time of discovering it. Dynamic Time Warping (DTW) based algorithms can only find a pattern of a specific length since it performs end-to-end matching of the query and test sequence. There is another version of DTW known as

Unconstrained End Point-DTW (UE-DTW) that can match the whole query with a partial test but still the query is not partially matched. Longest Common Subsequence (LCS) algorithm on the other hand can match the partial query with partial test sequence since it looks for a longest common subsequence which need not be end-to-end. LCS by itself is not appropriate as it requires discrete symbols and does not account for local similarity. A modified version of LCS known as Rough Longest Common Subsequence takes continuous symbols and takes into account the local similarity of the longest common subsequence. The algorithm proposed in [13] to find rough longest common sequence between two sequences fits the bill for our task of motif discovery. An example of RLCS algorithm matching two partial phrases is shown in Figure 1. The two music segments are represented by their tonic normalized smoothed pitch contours [9, 10]. The stationary points, where the first derivative is zero, of the tonic normalized pitch contour are first determined. The points are then interpolated using cubic Hermite interpolation to smooth the contour.

In previous uses of RLCS for motif spotting task [9, 10], a number of false alarms were observed. One of the most prevalent false alarms is the test phrase with a sustained note which comes in between the notes of the query. The slope of the linear trend in stationary points along with its standard deviation is used to address this issue.

The rest of the paper is organized as follows. In Section 2 the use of one-liners of compositions to find motifs is discussed. Section 3 discusses the optimization criteria to find the rough longest common subsequence. Section 4 describes the proposed approach for discovering typical motifs of *rāga*s. Section 5 describe the dataset used in this work. Experiments and results are presented in Section 6.

2. ONE-LINERS OF SONGS

As previously mentioned, first line of the composition contains the characteristic traits of a *rāga*. The importance of the first lines and the *rāga* information it holds is illustrated in great detail in the T. M. Krishna's book on Carnatic music [12]. T. M. Krishna states that opening section called "pallavi" directs the melodic flow of the *rāga*. Through its rendition, the texture of the *rāga* can be felt. Motivated by this observation, an attempt is made to verify the conjecture that typical motifs of a *rāga* can be obtained from the first lines of compositions.

Along with the lines from pallavi, we have also selected few lines from other sections, namely, 'anupallavi' and 'charanam'. Anupallavi comes after pallavi and the melodic movements in this section tend to explore the *rāga* in the higher octave [12]. These lines are referred to as one-liners for a *rāga*.

3. OPTIMIZATION CRITERIA TO FIND ROUGH LONGEST COMMON SUBSEQUENCE

The rough longest common subsequence (rlcs) between two sequences, $X = \langle x_1, x_2, \cdots, x_n \rangle$ and $Y = \langle y_1, y_2, \cdots$
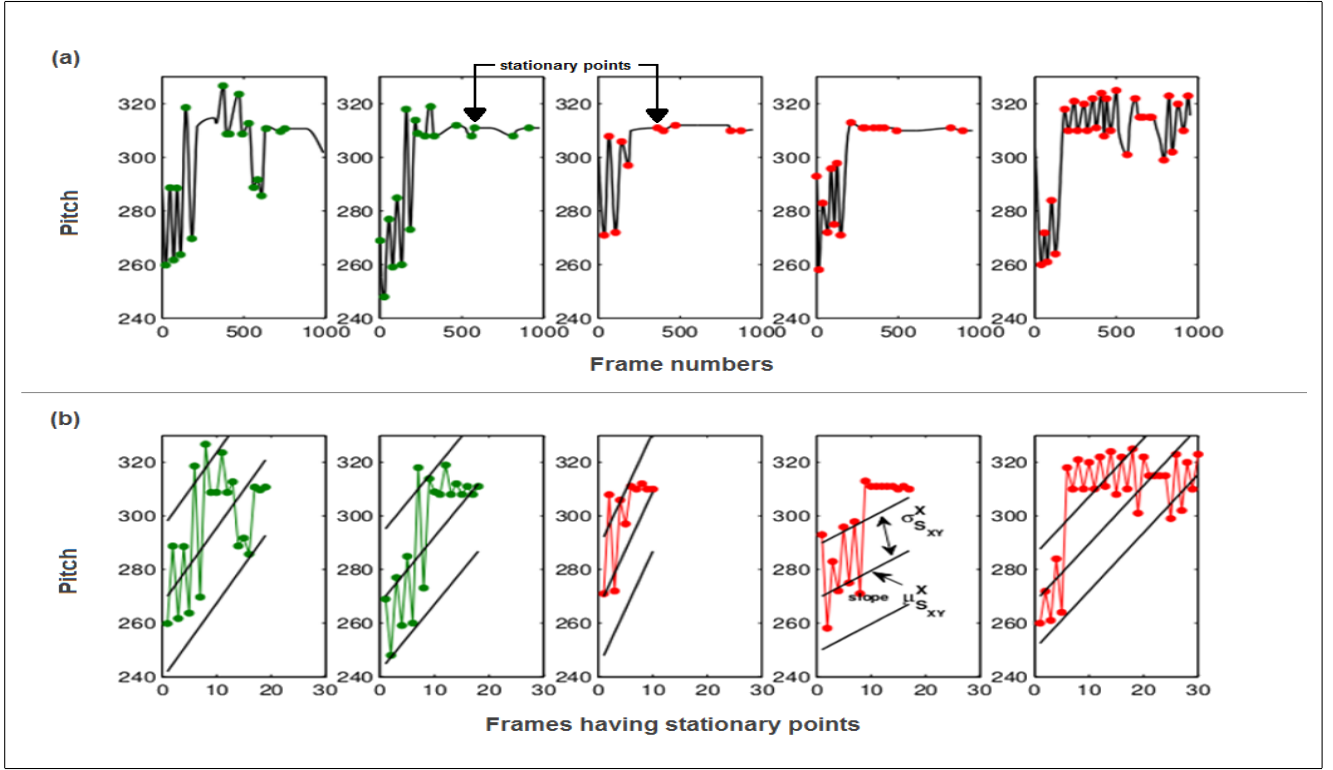
Figure 2. (a) Pitch contour of the five phrases which are considered similar. Stationary points are marked in green and red for the true positives and false alarms respectively. (b) Pitch values only at the stationary points. Slope of the linear trend in stationary points along-with its standard deviation helps in reducing the false alarms.

$, y_m \rangle$, of length n and m is defined as the longest common subsequence (lcs) $Z_{XY} = \langle (x_{i_1}, y_{j_1}), (x_{i_2}, y_{j_2}), \cdots, (x_{i_p}, y_{j_p}) \rangle, 1 \le i_1 < i_2 < \cdots < i_p \le n, 1 \le j_1 < j_2 < \cdots < j_p \le m$; such that the similarity between x_{i_k} and y_{j_k} is greater than a threshold, τ_{sim}, for $k = 1, \cdots, p$. There are no constraints on the length and on the local similarity of the rlcs. Some applications demand the rlcs to be locally similar or its length to be in a specific range. For the task of motif discovery along with these constraints, one more constraint is used to reduce false alarms. Before discussing the optimization measures used to find the rlcs in this work, a few quantities need to be defined.

$$l_{S_{XY}}^w = \sum_{k=1}^{s} sim(x_{i_k}, y_{j_k}) \tag{1}$$

$$g_X = i_s - i_1 + 1 - s \tag{2}$$

$$g_Y = j_s - j_1 + 1 - s \tag{3}$$

Let $S_{XY} = \langle (x_{i_1}, y_{j_1}), (x_{i_2}, y_{j_2}), \cdots, (x_{i_s}, y_{j_s}) \rangle, 1 \le i_1 < i_2 < \cdots < i_s \le n, 1 \le j_1 < j_2 < \cdots < j_s \le m$; be a rough common subsequence (rcs) of length s and $sim(x_{i_k}, y_{i_k}) \in [0, 1]$ be the similarity between x_{i_k} and y_{i_k} for $k = 1, \cdots, s$. Equation (1) defines the weighted length of S_{XY} as sum of similarities, $sim(x_{i_k}, y_{i_k})$, $k = 1, \cdots, s$. Thus, weighted length is less than or equal to s. The number of points in the shortest substring of sequence X, containing the rcs S_{XY}, that are not the part of the rcs S_{XY} are termed as gaps in S_{XY} with respect to sequence X as defined by Equation (2). Similarly, Equation (3) de-

fines the gaps in S_{XY} with respect to sequence Y. Small gaps indicate that the distribution of rcs is dense in that sequence.

The optimization measures to find the rlcs are described as follows.

3.1 Density of the match

Equation (4) represents the distribution of the rcs S_{XY} in the sequences X and Y. This is called density of match, $\delta_{S_{XY}}$. This quantity needs to be maximized to make sure the subsequence, S_{XY}, is locally similar. $\beta \in [0, 1]$ weighs the individual densities in sequences X and Y.

$$\delta_{S_{XY}} = \beta \frac{l_{S_{XY}}^w}{l_{S_{XY}}^w + g_X} + (1 - \beta) \frac{l_{S_{XY}}^w}{l_{S_{XY}}^w + g_Y} \tag{4}$$

3.2 Normalized weighted length

The weighted length of rcs is normalized as shown in Equation (5) to restrict its range to $[0, 1]$. n and m are the lengths of sequences X and Y, respectively.

$$\hat{l}_{S_{XY}}^w = \frac{l_{S_{XY}}^w}{\min(m, n)} \tag{5}$$

3.3 Linear trend in stationary points

As observed in [9, 10], the rlcs obtained using only the above two optimization measures suffered from a large number of false alarms for the motif spotting task. The false alarms generally constituted of long and sustained notes.

This resulted in good normalised weighted lengths and density. To address this issue, the slope and standard deviation of the slope of the linear trend in stationary points of a phrase are estimated. Figure 2 shows a set of phrases. This set has five phrases which are termed as similar phrases based on their density of match and normalized weighted length. The first two phrases, shown in green, are true positives while the remaining, shown in red, are false alarms. Figure 2 also shows the linear trend in stationary points for the corresponding phrases. It is observed that the trends are similar for true positives when compared to that of the false alarms. The slope of the linear trend for the fifth phrase (false alarm) is similar to the true positives but its standard deviation is less. Therefore, a combination of the slope and the standard deviation of the linear trend is used to reduce the false alarms.

Let the stationary points in the shortest substring of sequences X and Y containing the rcs S_{XY} be $\langle x_{q_1}, x_{q_2}, \cdots, x_{q_{t_x}} \rangle$ and $\langle y_{r_1}, y_{r_2}, \cdots, y_{r_{t_y}} \rangle$ respectively, where t_x and t_y are the number of stationary points in the respective substrings. Equation (6) estimates the slope of the linear trend, of stationary points in the substring of sequence X, as the mean of the first difference of stationary points, which is same as $\frac{x_{q_{t_x}} - x_{q_1}}{t_x - 1}$ [8]. Its standard deviation is estimated using Equation (7). Similarly, $\mu_{S_{XY}}^Y$ and $\sigma_{S_{XY}}^Y$ are also estimated for substring of sequence Y.

$$\mu_{S_{XY}}^X = \frac{1}{t_x - 1} \sum_{k=1}^{t_x - 1} (x_{q_{k+1}} - x_{q_k}) \tag{6}$$

$$\sigma_{S_{XY}}^X{}^2 = \frac{1}{t_x - 1} \sum_{k=1}^{t_x - 1} ((x_{q_{k+1}} - x_{q_k}) - \mu_{S_{XY}}^X)^2 \tag{7}$$

Let $z_1 = \mu_{S_{XY}}^X \sigma_{S_{XY}}^Y$ and $z_2 = \mu_{S_{XY}}^Y \sigma_{S_{XY}}^X$. For a true positive, the similarity in the linear trend should be high. Equation (8) calculates this similarity which needs to be maximized. This similarity has negative value when the two slopes are of different sign and thus, the penalization is more.

$$\rho_{S_{XY}} = \begin{cases} \frac{\max(z_1, z_2)}{\min(z_1, z_2)} & if \ z_1 < 0; z_2 < 0 \\[2mm] \frac{\min(z_1, z_2)}{\max(z_1, z_2)} & otherwise \end{cases} \tag{8}$$

Finally, Equation (9) combines these three optimization measures to get a score value which is maximized. Then the rlcs, R_{XY}, between the sequences X and Y is defined, as an rcs with a maximum score, in Equation (10). The rlcs R_{XY} can be obtained using dynamic programming based approach discussed in [9, 13].

$$Score_{S_{XY}} = \alpha \delta_{S_{XY}} \hat{l}_{S_{XY}}^w + (1 - \alpha)\rho_{S_{XY}} \tag{9}$$

$$R_{XY} = \underset{S_{XY}}{\operatorname{argmax}} (Score_{S_{XY}}) \tag{10}$$

Rāga Name	Number of one-liners	Average duration (secs)
Bhairavi	17	16.87
Kamboji	12	13
Kalyani	9	12.76
Shankarabharanam	12	12.55
Varali	9	9.40
Overall	59	12.91

Table 1. Database of one-liners

4. DISCOVERING TYPICAL MOTIFS OF *RĀGAS*

Typical motifs of a *rāga* are discovered using one-liners of songs in that *rāga*. For each voiced part in a oneliner of a *rāga*, rlcs is found with the overlapping windows in voiced parts of other one-liners of that *rāga*. Only those rlcs are selected whose score values and lengths (in seconds) are greater than thresholds τ_{scr} and τ_{len} respectively The voiced parts which generated no rlcs are interpreted to have no motifs. The rlcs generated for a voiced part are grouped and this group is interpreted as a motif found in that voiced part. This results in a number of groups (motifs) for a *rāga*. Further, filtering is performed to isolate typical motifs of that *rāga*.

4.1 Filtering to get typical motifs of a *rāga*

The generated motifs are filtered to get typical motifs of a *rāga* using compositions of various *rāga*s. The most representative candidate of a motif, a candidate with highest score value, is selected to represent that motif or group. The instances of a motif are spotted in the compositions of various *rāga*s as explained in [9,10]. Each motif is considered as a query to be searched for in a composition. The rlcs is found between the query and overlapped windows in a composition. From the many generated rlcs from many compositions of a *rāga*, top τ_n rlcs with highest score values are selected. The average of these score values defines the presence of this motif in that *rāga*. A motif of a *rāga* is isolated as its typical motif if the presence of this motif is more in the given *rāga* than in other *rāga*s. The value of τ_n is selected empirically.

5. DATASET

The one-liners are selected from five *rāga*s as shown in Table 1. The lines are sung by a musician in isolation. This is done to ensure that the pitch estimation does not get affected due to the accompanying instruments. The average duration of the one-liners is 12.91 seconds. As mentioned earlier, these one-liners come from the various sections of the composition, primarily from the pallavi.

The compositions used for filtering also comes from the same five *rāga*s as shown in Table 2. These compositions are taken from the Charsur collection [1]. These are segments from live concerts with clean recording.

Rāga Name	Number of compositions	Average duration (secs)
Bhairavi	20	1133
Kamboji	10	1310.3
Kalyani	16	1204.3
Shankarabharanam	10	1300.6
Varali	18	1022
Overall	74	1194

Table 2. Database of compositions

Rāga Name	Number of discovered patterns	Average duration (secs)
Bhairavi	10	3.52
Kamboji	5	3.40
Kalyani	6	4.48
Shankarabharanam	6	3.42
Varali	3	3.84
Overall	30	3.73

Table 3. Summary of discovered similar patterns across one-liners

6. EXPERIMENTS AND RESULTS

The pitch of the music segment is used as a basic feature in this work. This pitch is estimated from Justin Solomon's algorithm [17] which is efficiently implemented in the essentia open-source C++ library [2]. This pitch is further normalized using tonic and then smoothed by interpolating the stationary points of the pitch contour using cubic spline interpolation.

The similarity, $sim(x_{i_k}, y_{j_k})$, between two symbols x_{i_k} and y_{j_k} is defined in the Equation (11), where s_t is the number of cent values that represent one semitone. For this work, the value of s_t is 10. The penalty is low when the two symbols are within one semitone while the penalty is significant for larger deviations. This is performed to ensure that although significant variations are possible in Carnatic music, variations larger than a semitone might result in a different *rāga*.

$$sim(x_{i_k}, y_{j_k}) = \begin{cases} 1 - \frac{|x_{i_k} - y_{j_k}|^3}{(3s_t)^3} & if \ |x_{i_k} - y_{j_k}| < 3s_t \\ 0 & otherwise \end{cases}$$
(11)

The similarity threshold, τ_{sim}, is empirically set to 0.45 which accepts similarities when two symbols are less than 2.5 semitones (approx.) apart, although penalty is high after a semitone. The threshold on the score of rlcs, τ_{scr}, is empirically set to 0.6 to accept rlcs with higher score values. The threshold on the length of the rlcs, τ_{len}, is set to 2 seconds to get longer motifs. The value of β is set to 0.5 to give equal importance to the individual densities in both the sequences and α value is set to 0.6 which gives more importance to density of match and normalized weighted length as compared to linear trend in stationary points. τ_n is empirically set to 3.

The similar patterns found across one-liners of a *rāga* are summarized in Table 3. Some of these similar patterns are not typical of the *rāga*. These are therefore filtered out by checking for their presence in various compositions. The summary of the resulting typical motifs is given in Table 4. The average length of all the typical motifs is sufficiently longer than what were used in [10]. The shorter motifs used in [10] also resulted in great deal of false alarms. The importance of longer motifs was discussed in [9] where the longer motifs were inspired from the *rāga* test conducted by Rama Verma [3]. Rama Verma

Rāga Name	Number of typical motifs	Average duration (secs)
Bhairavi	5	4.52
Kamboji	0	NA
Kalyani	0	NA
Shankarabharanam	5	3.64
Varali	2	4.79
Overall	12	4.32

Table 4. Summary of typical motifs isolated after filtering

used motifs of approximately 3 seconds duration. The typical motifs discovered in our work are also of similar duration. All the patterns of Kamboji and Kalyani are filtered out resulting in no typical motifs for these *rāga*s. We have earlier discussed that the compositions in Carnatic music are composed in a way that the *rāga* information is present at the very beginning. Therefore, without a doubt we are sure that the typical motifs are present in the one-liners we have used for Kalyani and Kamboji. But, it is possible that these typical motifs are not repeating sufficient number of times across one-liners (two times in our approach) or their lengths are shorter than the threshold we have used. These could be the reasons we are not able to pick them up. All the typical patterns are verified by a musician. According to his judgment, all the filtered patterns were indeed typical motifs of the corresponding *rāga*s. Although, he noted that one typical motif in Varali is a smaller portion of the other discovered typical motif of Varali. This repetition of smaller portion is observed in Shankarabharanam as well.

7. CONCLUSION AND FUTURE WORK

This paper presents an approach to discover typical motifs of a *rāga* from the one-liners of the compositions in that *rāga*. The importance of one-liners is discussed in detail. A new measure is introduced, to reduce the false alarms, in the optimization criteria for finding rough longest common subsequence between two given sequences. Using the RLCS algorithm, similar patterns across one-liners of a *rāga* are found. Further, the typical motifs are isolated by a filtering technique, introduced in this paper, which uses compositions of various *rāga*s. These typical motifs

are validated by a musician. All the generated typical motifs are found to be significantly typical of their respective *rāga*s.

In this work, only one musician's viewpoint is considered on validating the characteristic nature of the discovered typical motifs. In future, we would like to conduct a MOS test, asking other experts and active listeners to determine the *rāga* from the typical motifs. We would also like to perform *rāga* classification of the compositions and alapanas using the typical motifs. In future, we would also like to do a thorough comparison of our approach with other methods. In this paper, we have only addressed one prevalent type of false alarms. Other types of false alarms also need to be identified and addressed. It should be considered that approaches taken to reduce the false alarms do not affect the true positives significantly. Further, these experiments need to be repeated for a much larger number of one-liners from many *rāga*s such that the typical motifs repeat significantly across one-liners and thus get captured. It will also be interesting to automatically detect and extract the one-liners from the available compositions. This will enable the presented approach to scale to a large number of *rāga*s.

8. ACKNOWLEDGMENTS

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