BURSTINESS AND HIERARCHY IN TONAL CLASSICAL MUSIC

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ABSTRACT

A musical work is, in general, a coherent whole that is more than the sum of its individual notes. We are interested in the emergent coherent behavior that arises from the combination of the sounds made by the individual pitches, focusing on the temporal aspect. We consider each pitch activation as an event, and examine the distribution of the interevent times, the time between the deactivation and consecutive activation of the same pitch. The interevent times in a sample of 428 works by four canonical Western composers obey heavy-tailed distributions, and these distributions can be attributed not to the pitch durations themselves but to the order in which pitches are activated. Our results imply that the generative process in music is neither random nor regular, and suggest the presence of hierarchy in pitch activations. We present our initial attempt in creating a hierarchical generative model of music, inspired by the similarities between our findings and Schenkerian analysis.

1. INTRODUCTION

Consider a sequence of events in time. Some examples are the arrival of trains in a station, the calls made by a person, or in our study, the activation of pitches in a musical piece. The patterns of the event occurrence can be characterized by the time between events, also called the "interevent time". If the events are regular, then the interevent time is constant. If the events are random, the interevent times will be uncorrelated and follow an exponential distribution. However, in the case of a bursty sequence of events, most events happen close to each other in time, while some events are spaced far apart from each other. This creates clusters of events or "bursts", and the interevent time distribution is heavy-tailed, usually following a power law [1]. These indicate that in contrast to random events, the events in a bursty sequence do not happen independently and suggest a long-range correlation over time, possibly induced by a hierarchy in the sequence of events [2–4]. If music exhibits bursty behavior, burstiness may be used to develop a data-driven approach to find hierarchical structure in music, as posited by researchers such as Schenker [5], Lerdahl and Jackendoff [6], Krumhansl [7], and Narmour [8].

2. METHODOLOGY

We use the `chordify` function of the Python package `music21` [9] to split a piece into a series of vertical slices, where each slice corresponds to a change in the texture by adding or subtracting a pitch. By chordifying MIDI files from KernScores, we extract the activation sequence of each pitch from 428 pieces by Bach, Beethoven, Mozart and Chopin, chosen as prominent composers of their times and with a substantial contribution to the KernScores corpus. Using the quarter note as our unit of time, we obtain the times between pitch deactivation ("off" in MIDI) to pitch reactivation ("on" in MIDI) for each pitch. These constitute our interevent times (IETs), with the events being pitch activations. We fit the IET distributions to various functions (exponential, lognormal, stretched exponential, and truncated power law) using the algorithm presented by Clauset et al. [10, 11] and select the distribution that gives the best fit. To identify the ingredients that are essential for the IET distributions, we modify the original piece in different ways: (a) `ordinaltime`, where the slices are made to have equal duration, effectively disregarding note duration; (b) `ordinalpp`, similar to `ordinaltime` but with consecutive activations considered as one and the note duration set to 0 (the IET is effectively the pitch interonset interval); (c) `topnote_ordinaltime`, an approximation of the melodic line in `ordinaltime`, and (d) `randomnote88`, where the note durations are retained but the pitches in each slice are replaced with a random selection from the 88 pitches of a piano, thus destroying tonality in the piece.

3. RESULTS

The IET distributions of the original piece as well as `ordinaltime`, `ordinalpp` and `topnote_ordinaltime` were mostly heavy-tailed indicating bursty behavior (see Figure 1), with only 4 out of the 428 pieces exhibiting exponentially distributed

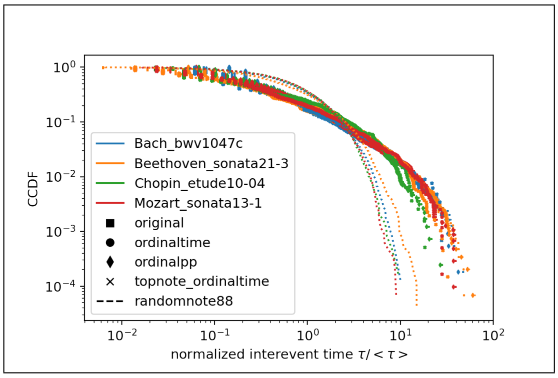

Figure 1. Complementary cumulative distribution functions of the interevent time distributions normalized by their means for four different pieces, with the composer and the piece title indicated in the legend. The distributions for the original piece and the variants `ordinaltime`, `ordinalpp` and `topnote_ordinaltime` are differentiated by the marker symbols, while `randomnote88` is shown using dashed lines. Note that all the curves denoted by symbols obey a similar shape distinct from that of the dashed curves.

IETs. In contrast, 415 of `randomnote88` pieces had either exponential distributions or stretched exponential distributions, with the mean exponent in stretched exponential distributions of 0.96. Further, the IET distributions of different pieces obey similar shapes despite having been from different composers and different eras. These findings show that pitch activations are bursty and correlated, and that there are commonalities in the activation patterns of pitches in various pieces of classical tonal music. Further, the distribution of the IETs divided by the mean are similar for the original piece and `ordinaltime`, `ordinalpp` and `topnote_ordinaltime`, indicating that the note duration doesn't contribute to burstiness. However, the rules of melody and harmony, which are absent in `randomnote88`, do. We note here some similarities to Schenkerian analysis, where note durations are disregarded and only the sequence of pitches is considered [5, 12]. We also note that the burstiness we observed is not in relation to the general rhythm of the piece but to the time between deactivation and activation of the *same* pitch.

4. THE MODEL

Bursty behavior can be explained by a number of mechanisms, including hierarchy. Given the similarities of our results to aspects of Schenkerian analysis, we attempt to explain burstiness in music using a hierarchical generative model.

We initialize our model with a starting sequence of pitches of length N_0. Starting from the first two elements of the sequence, we splice every pair of adjacent pitches (P_i, P_{i+1}) and insert one new pitch. The inserted pitch is selected from the set of pitches in the original melody that succeed P_i and precede P_{i+1} with a probability that is proportional to the sum of the number of times it succeeded P_i and the number of times it preceded P_{i+1}. We iterate until the desired length of the sequence is reached. If $N_0 = 1$, a pitch that succeeds the lone sequence element in the melody will be placed after it with a probability proportional to the number of times the candidate pitch succeeded the lone sequence element. Once there are two pitches in the sequence, the iterations proceed as normal. The order of splicing and insertions denotes the hierarchy, as the pitches that were inserted later can be thought of as embellishments to the preceding pitches.

The model generates heavy-tailed distributions (see Figure 2) that are due to the inherent correlations in the sequence of pitches, which is incorporated in the model in the insertion procedure. We note however that although the qualitative results are similar, there are quantitative discrepancies in the model results and the `topnote_ordinaltime` distributions. In particular, we get more longer interevent times with the model results than the original pieces.

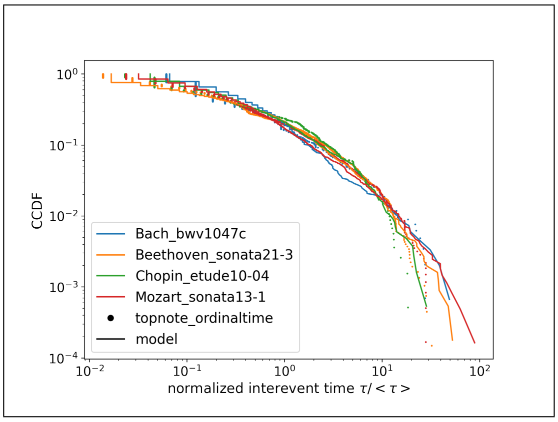

Figure 2. Complementary cumulative distribution functions of the interevent time distributions normalized by their means for four different pieces, with the composer and the piece title indicated in the legend. The distributions for `topnote_ordinaltime` and the model ($N_0 = 1$) results are shown.

5. CONCLUSION

Canonical Western classical composers representing Baroque through Romantic eras are found to exhibit bursty behavior that depends only on the sequence of notes and not their durations, indicating correlations in pitch activations. Noting the similarities of our results with some aspects of cognitive theories of musical hierarchies, we construct a hierarchical model based on iterative embellishments to recreate the bursty behavior obtained. Although our model exhibits burstiness, it gives longer interevent times than the original pieces. We hope to improve our model by allowing transitions outside those used in the actual piece.

6. REFERENCES

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